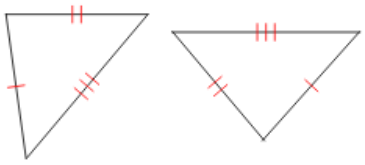
Unit 4 Lesson 6 - Proving Congruence in Triangles

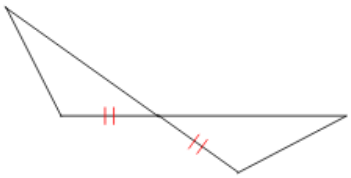
Recall triangle congruence postulates:

- SIDE SIDE SIDE (SSS) Congruence – If three sides of one triangle are congruent to three sides of a second triangle, then the triangles are congruent.
- SIDE ANGLE SIDE (SAS) Congruence – If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the triangles are congruent.
- ANGLE SIDE ANGLE (ASA) Congruence – If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.
- ANGLE ANGLE SIDE (AAS) Congruence – If two angles and the non-included side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the two triangles are congruent.

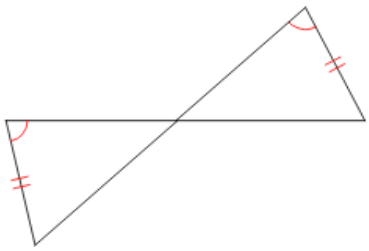
Determine whether the two triangles below are congruent. If they are, which postulate above proves congruence:

1) 

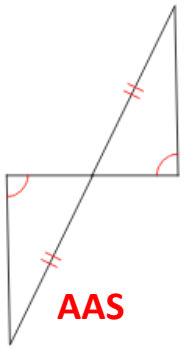
SSS

2) 

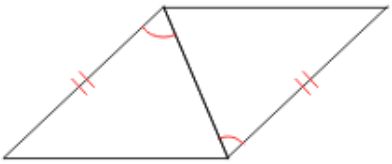
NOT ≅

3) 

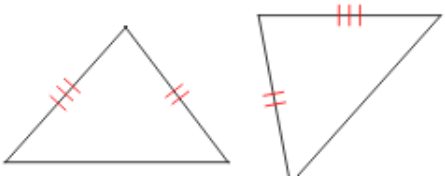
AAS

4) 

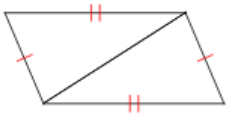
AAS

5) 

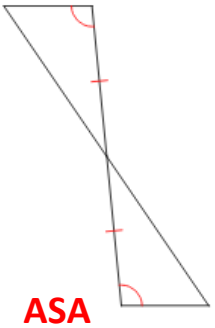
SAS

6) 

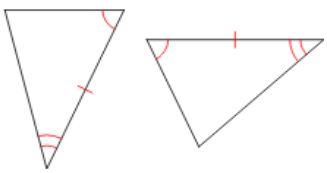
NOT ≅

7) 

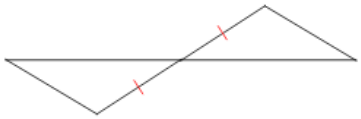
SSS

8) 

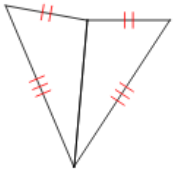
ASA

9) 

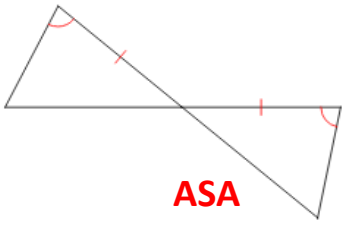
ASA

10) 

NOT ≅

11) 

SSS

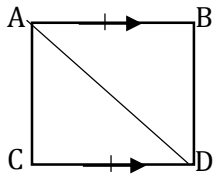
12) 

ASA

Proving Triangles Congruent

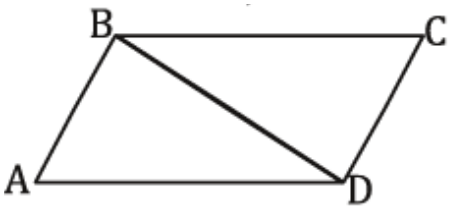
- Reflexive Property of Triangle Congruence → $\Delta ABC \cong \Delta ABC$
- Symmetric Property of Triangle Congruence → If $\Delta ABC \cong \Delta EFG$, then $\Delta EFG \cong \Delta ABC$

Given the figure below, prove that $\Delta ACD \cong \Delta CAB$.



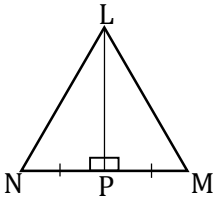
Statement	Reason
1. $AB = CD, \overline{AB} \parallel \overline{CD}$	GIVEN
2. $\angle BAD \cong \angle ADC$	ALTERNATE INTERIOR ANGLES
3. $AD = AD$	REFLEXIVE PROPERTY
4. $\Delta ACD \cong \Delta ADB$	SAS

Given $\overline{AB} \cong \overline{CD}, \overline{AD} \cong \overline{CB}$, prove $\Delta ABD \cong \Delta CBD$.



Statement	Reason
1. $\overline{AB} \cong \overline{CD}$	GIVEN
2. $\overline{AD} \cong \overline{CB}$	GIVEN
3. $\overline{BD} \cong \overline{BD}$	REFLEXIVE PROPERTY
4. $\Delta ABD \cong \Delta CBD$	SSS

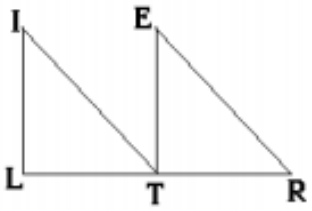
Given the figure below, prove that $\Delta NPL \cong \Delta MPL$.



Statement	Reason
1. $NP = PM, \overline{NP} \perp \overline{PL}$	GIVEN
2. $\angle MPL$ is a right angle $\angle NPL$ is a right angle	GIVEN, RIGHT ANGLES ARE CONGRUENT
3. $PL = PL$	REFLEXIVE PROPERTY
4. $\Delta NPL \cong \Delta MPL$	SAS

Writing proofs without statements:

1. Start with the given information.
2. Fill in properties/theorems you can infer.
3. End with what you are trying to prove.



Given: $\overline{LT} \cong \overline{TR}, \angle ILT \cong \angle ETR, \overline{IT} \parallel \overline{ER}$

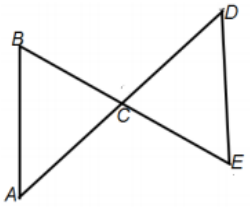
Prove: $\Delta LIT \cong \Delta TER$

Statement	Reason
1. $\overline{LT} \cong \overline{TR}$	GIVEN
2. $\angle ILT \cong \angle ETR$	GIVEN
3. $\overline{IT} \parallel \overline{ER}$	GIVEN
4. $\angle ERT \cong \angle ITL$	CORRESPONDING ANGLES ARE CONGRUENT
5. $\Delta LIT \cong \Delta TER$	ASA

Given: $\overline{BA} \cong \overline{ED}$

C is the midpoint of $\overline{BE}$ and $\overline{AD}$

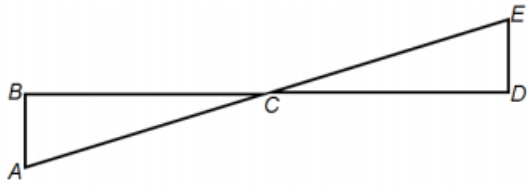
Prove: $\Delta ABC \cong \Delta DEC$



Statement	Reason
1. $\overline{BA} \cong \overline{ED}$	GIVEN
2. $\overline{BC} \cong \overline{EC}$	DEFINITION OF MIDPOINT
3. $\overline{AC} \cong \overline{DC}$	DEFENITION OF MIDPOINT
4. $\angle BCA \cong \angle ECD$	VERTICAL ANGLES ARE CONGRUENT
5. $\Delta ABC \cong \Delta DEC$	SSS OR SAS

Given: C is the midpoint of $\overline{BD}$. $\overline{AB} \perp \overline{BD}, \overline{BD} \perp \overline{DE}$

Prove: $\Delta ABC \cong \Delta EDC$

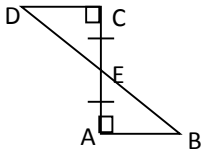


Statement	Reason
1. $\overline{BC} \cong \overline{DC}$	DEFINITION OF MIDPOINT
2. $\overline{AB} \perp \overline{BD}$	GIVEN
3. $\overline{BD} \perp \overline{DE}$	GIVEN
4. $\angle B$ IS A RIGHT $\angle$	DEFINITION OF PERPENDICULAR
5. $\angle D$ IS A RIGHT $\angle$	DEFINITION OF PERPENDICULAR
6. $\angle BCA \cong \angle DCE$	VERTICAL ANGLES ARE CONGRUENT
7. $\Delta ABC \cong \Delta EDC$	ASA

Practice Proofs

For each pair of triangles, tell: (a) Are they congruent (b) Write the triangle congruency statement. (c) Give the postulate that makes them congruent. Then write a prove in the bale provided.

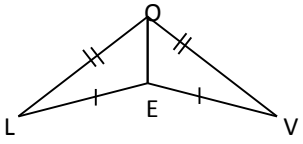
1.



a. YES
b. Δ DCE ≅ Δ BAE
c. ASA

Statement	Reason
$\angle C \cong \angle A$	GIVEN
$\overline{CE} \cong \overline{AE}$	GIVEN
$\angle CED \cong \angle AEB$	VERTICAL ∠
$\Delta DCE \cong \Delta BAE$	ASA

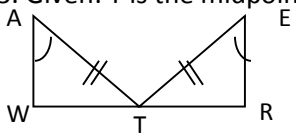
2.



a. YES
b. Δ OEL ≅ Δ OEV
c. SSS

Statement	Reason
$\overline{LO} \cong \overline{VO}$	GIVEN
$\overline{LE} \cong \overline{VE}$	GIVEN
$\overline{EO} \cong \overline{EO}$	REFLEXIVE
$\Delta OEL \cong \Delta OEV$	SSS

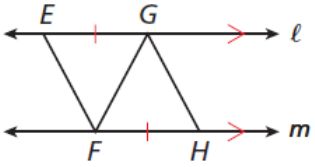
3. Given: T is the midpoint of WR



a. NO
b. Δ _____ ≅ Δ _____
c. _____

Statement	Reason

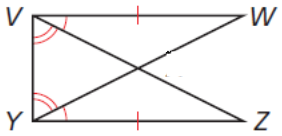
4.



a. YES
b. Δ EGF ≅ Δ HFG
c. SAS

Statement	Reason
$\overline{EG} \cong \overline{HF}$	GIVEN
$\angle EGF \cong \angle HFG$	ALT. INT.
$\overline{GF} \cong \overline{GF}$	REFLEXIVE
$\Delta EGF \cong \Delta HFG$	SAS

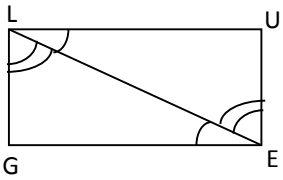
5.



a. YES
b. Δ VYW ≅ Δ YVZ
c. AAS

Statement	Reason
$\angle V \cong \angle Y$	GIVEN
$\angle Y \cong \angle V$	GIVEN
$\overline{VW} \cong \overline{YZ}$	GIVEN
$\Delta VYW \cong \Delta YVZ$	AAS

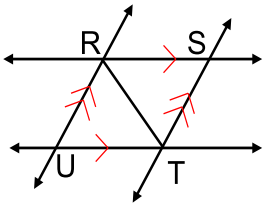
6.



a. YES
b. Δ ELG ≅ Δ LEU
c. ASA

Statement	Reason
$\angle ULE \cong \angle GEL$	GIVEN
$\overline{LE} \cong \overline{EL}$	REFLEXIVE
$\angle ELG \cong \angle LEU$	GIVEN
$\Delta ELG \cong \Delta LEU$	ASA

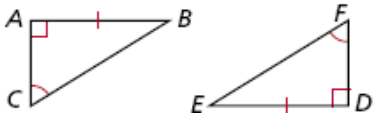
7.



a. NO
b. Δ _____ ≅ Δ _____
c. _____

Statement	Reason

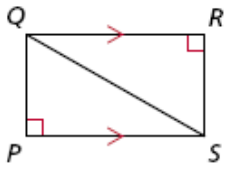
8.



a. YES
b. Δ ABC ≅ Δ DEF
c. AAS

Statement	Reason
$\angle A \cong \angle D$	GIVEN
$\angle C \cong \angle F$	GIVEN
$\overline{BA} \cong \overline{ED}$	GIVEN
$\Delta ABC \cong \Delta DEF$	AAS

9.



a. YES
b. Δ QRS ≅ Δ SPQ
c. AAS

Statement	Reason
$\angle R \cong \angle P$	GIVEN
$\angle RQS \cong \angle PSQ$	ALT. INT.
$\overline{QS} \cong \overline{SQ}$	REFLEXIVE
$\Delta QRS \cong \Delta SPQ$	AAS