Unit 2 Test Review: Exponential and Logarithmic Functions Name: ANSWER KEY

Unit 2 Learning Target 1: I can solve equations using logarithms. (____/20)

Find the value of x in each equation below. Show all of your work. Round to the nearest thousandth when necessary.

1) $60^{4x} = 831$

x = $.41$

$\log_{60} 831 = 4x$

$\frac{\log 831}{\log 60} = 4x$

2) $22^{2x-5} = 111$

x = 3.26

$\log_{22} 111 = 2x-5$

$\frac{\log 111}{\log 22} = 2x-5$

$1.524 = 2x-5$

$+5 \quad +5$

$\frac{6.524}{2} = \frac{2x}{2}$

3) $4^{x-1} + 3 = 57$

$-3 \quad -3$

x = 2.877

$4^{x-1} = 54$

$\log_4 54 = x-1$

$\frac{\log 54}{\log 4} = x-1$

$2.877 = x-1$

4) $-5 \cdot 2^{3x+1} = -55$

x = $.819$

$\frac{-5 \cdot 2^{3x+1}}{-5} = \frac{-55}{-5}$

$2^{3x+1} = 11$

$\log_2 11 = 3x+1$

$\frac{\log 11}{\log 2} = 3x+1$

$3.45 = 3x+1$

$-1 \quad -1$

$\frac{2.45}{3} = \frac{3x}{3}$

5) $1,000 is placed into an account where is gains 11% interest compounded annually. How long will it take the investment to double?

$\frac{2000}{1000} = \frac{1000}{1000}\left(1 + \frac{.11}{1}\right)^{(1t)}$

$2 = \left(1 + \frac{.11}{1}\right)^{1t}$

$\log_{(1+\frac{.11}{1})}(2) = t$

$\frac{\log 2}{\log(1+\frac{.11}{1})} = \boxed{6.64}$

Rewrite the following exponential functions as logarithms:

6) $25^{\frac{1}{2}} = 5$ $\log_{25} 5 = \frac{1}{2}$

7) $6^4 = 1296$ $\log_6 1296 = 4$

Rewrite the follow logarithms as exponential functions:

8) $\log_8 4096 = 4$ $8^4 = 4096$

9) $\log_{625} 5 = \frac{1}{4}$ $625^{\frac{1}{4}} = 5$

Unit 2 Learning Target 2: I can create an exponential model from context, and use it to solve problems.(____/20)

You have $500 to invest over the next 8 years at an interest rate of 6.5%. Find the total value of the investment for each type of interest below:

10)Compounded Annually:

a. Equation = $500\left(1+\frac{.065}{1}\right)^{(1\cdot 8)}$ Solution = \$827.49

11)Compounded Quarterly:

a. Equation = $500\left(1+\frac{.065}{4}\right)^{(4\cdot 8)}$ Solution = \$837.51

12)Compounded Monthly:

a. Equation = $500\left(1+\frac{.065}{12}\right)^{(12\cdot 8)}$ Solution = \$839.83

13)Compounded Continuous:

a. Equation = $500e^{.065\cdot 8}$ Solution = \$841.01

14)The population in Tanzania in 1987 was about 24.3 million, with an annual growth rate of 3.5%. The population is assumed to change continuously at this rate.

a. Write an equation to model the population of Tanzania: $y = 24.3e^{.035t}$

b. What was the population of Tanzania in the year 2008? $24.3e^{.035\cdot 21} = 50.67$ million

c. How long will it take for the population to reach 30,000,000?

$\frac{30}{24.3} = \frac{24.3e^{.035t}}{24.3}$

$1.2345 = e^{.035t}$

$LN_e(1.2345) = .035t$

$\frac{.2107}{.035} = \frac{.035t}{.035}$

$\boxed{6.02 = t}$

15)The table to the right models the number of rats living in an abandoned house. Create an equation to model the number of rats in the house after x years.

a) y = $50(3)^x$

b) Based on the equation created in part a, how long will it take the value of the table to reach 10,000?

x	y
0	50
1	150
2	450
3	1,350

$\frac{10,000}{50} = \frac{50(3)^x}{50}$

$200 = 3^x$

$\log_3 200 = x$

$\frac{\log 200}{\log 3} = x$

$\boxed{4.8 \text{ yrs}}$

16)The table to the right models the number of white rhinos living in Namibia. Let x = the # of years since 2000. Create an equation to model the white rhino population.

Year	# of Rhinos
2000	500
2002	442
2005	367
2009	287

a) y = $499.94(.94)^x$

b) Based on the equation created in part a, how long will it take the population to reach half the original population?

$\frac{250}{499.94} = \frac{499.94(.94)^x}{499.94}$ $(.94)^x = .5$ $\log_{.94}(.5) = x$ $\frac{\log(.5)}{\log(.94)} = \boxed{x = 11.2 \text{ yrs}}$

Unit 2 Learning Target 3: I can identify and explain the meaning of parts of an exponential equation in context. (___/10)

Determine whether each equation below represents growth or decay, and then identify the percent of increase or decrease.

17. $y = 2{,}500(0.92)^x$ GROWTH or DECAY by 8 %

18. $f(x) = 400(1.045)^x$ GROWTH or DECAY by 4.5 %

19. $h(x) = 50(0.60)^x$ GROWTH or DECAY by 40 %

20. The population of Mexico can be modeled by the equation $P = 107.4(1.012)^t$ where P is the total population (in millions) after t years after 2006.

a. Does this equation model growth or decay? Growth

b. By what percent did the population increase or decrease by each year? 1.2%

c. What was the population on Mexico in 2006? 107.4

21. The amount a car is worth can be modeled by the equation $y = 25{,}000(.825)^x$ where y represents the cost of the car x years after 2012.

a. Does this equation model growth or decay? Decay

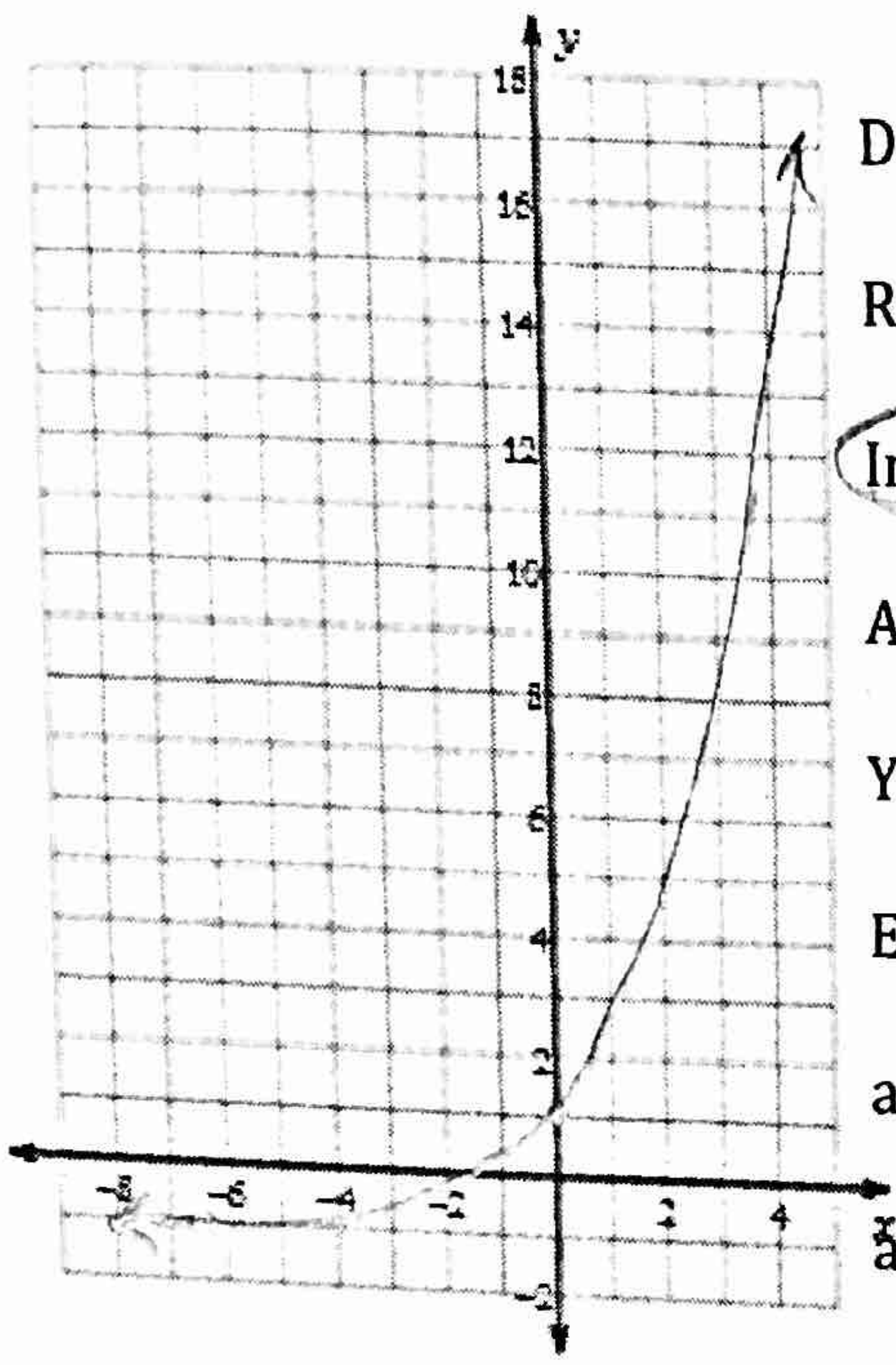
b. By what percent did the cars worth increase or decrease by each year? 17.5%

c. What was the car worth in 2012? 25,000

Unit 2 Learning Target 4: I can graph exponential and logarithmic functions and identify key features of the graphs, including transformations. (___/20)

For each equation below, create a graph and identify the key features.

22. $f(x) = 2^{x+1} - 1$



Domain: $(-\infty, \infty)$

Range: $(-1, \infty)$

Increase or Decrease

Asymptote @ -1

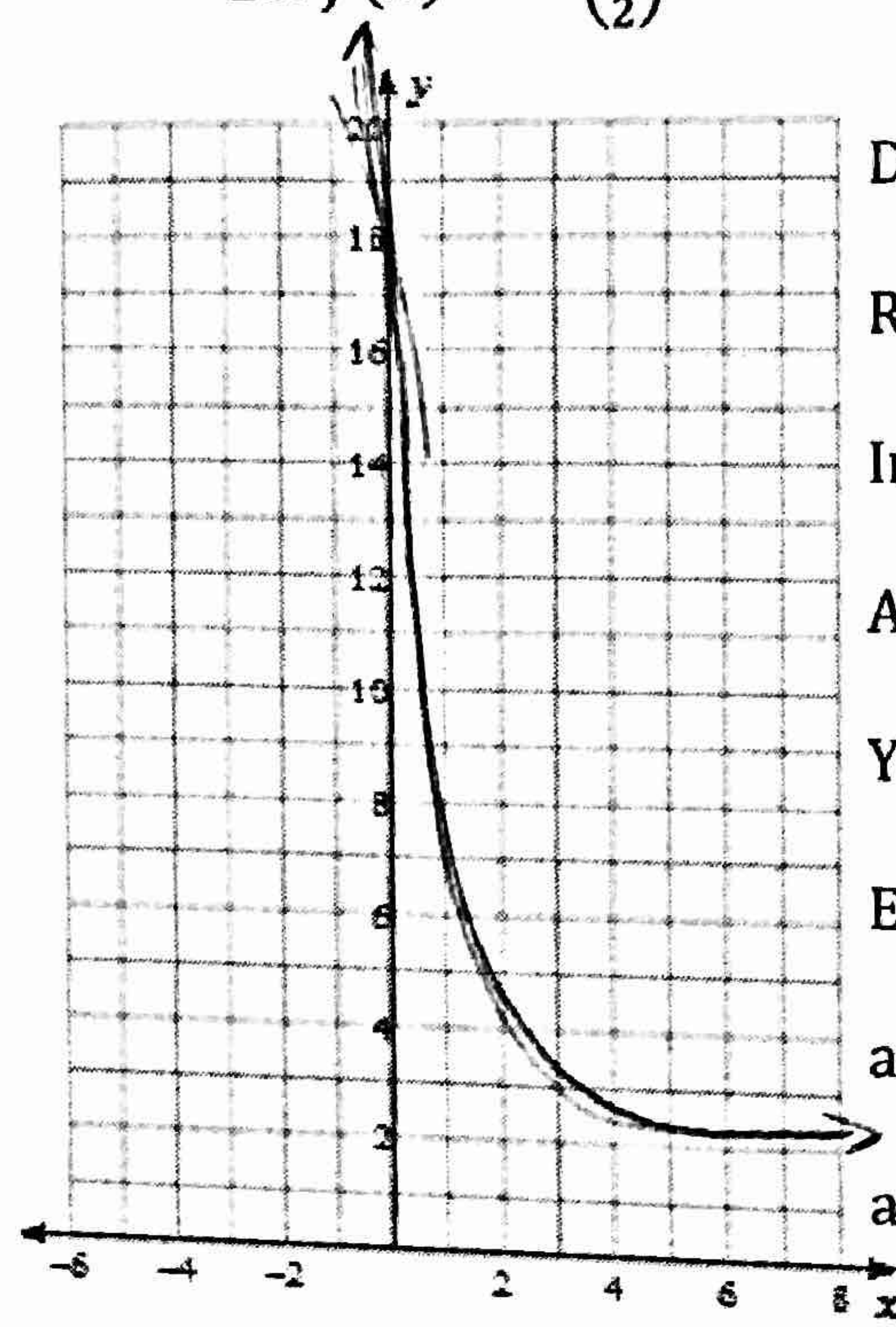
Y-Intercept @ 1

End behavior

as $x \to \infty, y \to$ ∞

as $x \to -\infty, y \to$ -1

23. $f(x) = 4\left(\frac{1}{2}\right)^{x-2} + 2$



Domain: $(-\infty, \infty)$

Range: $(2, \infty)$

Increase or Decrease

Asymptote @ 2

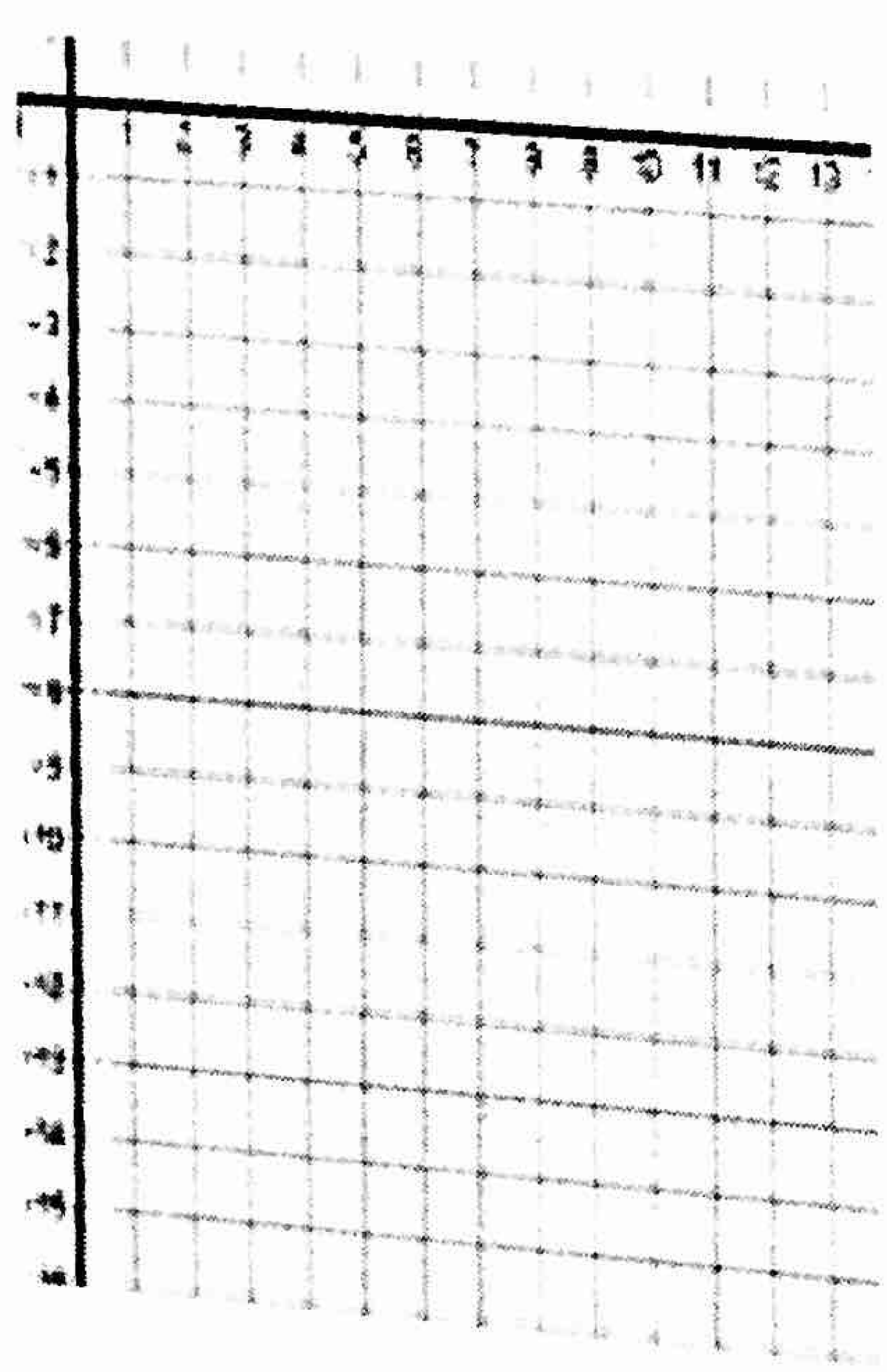
Y-Intercept @ 18

End behavior

as $x \to \infty, y \to$ -2

as $x \to -\infty, y \to$ ∞

24. $f(x) = log(x - 2) - 4$



Domain: $(2, \infty)$

Range: $(-\infty, \infty)$

Increase or Decrease

Asymptote @ 2

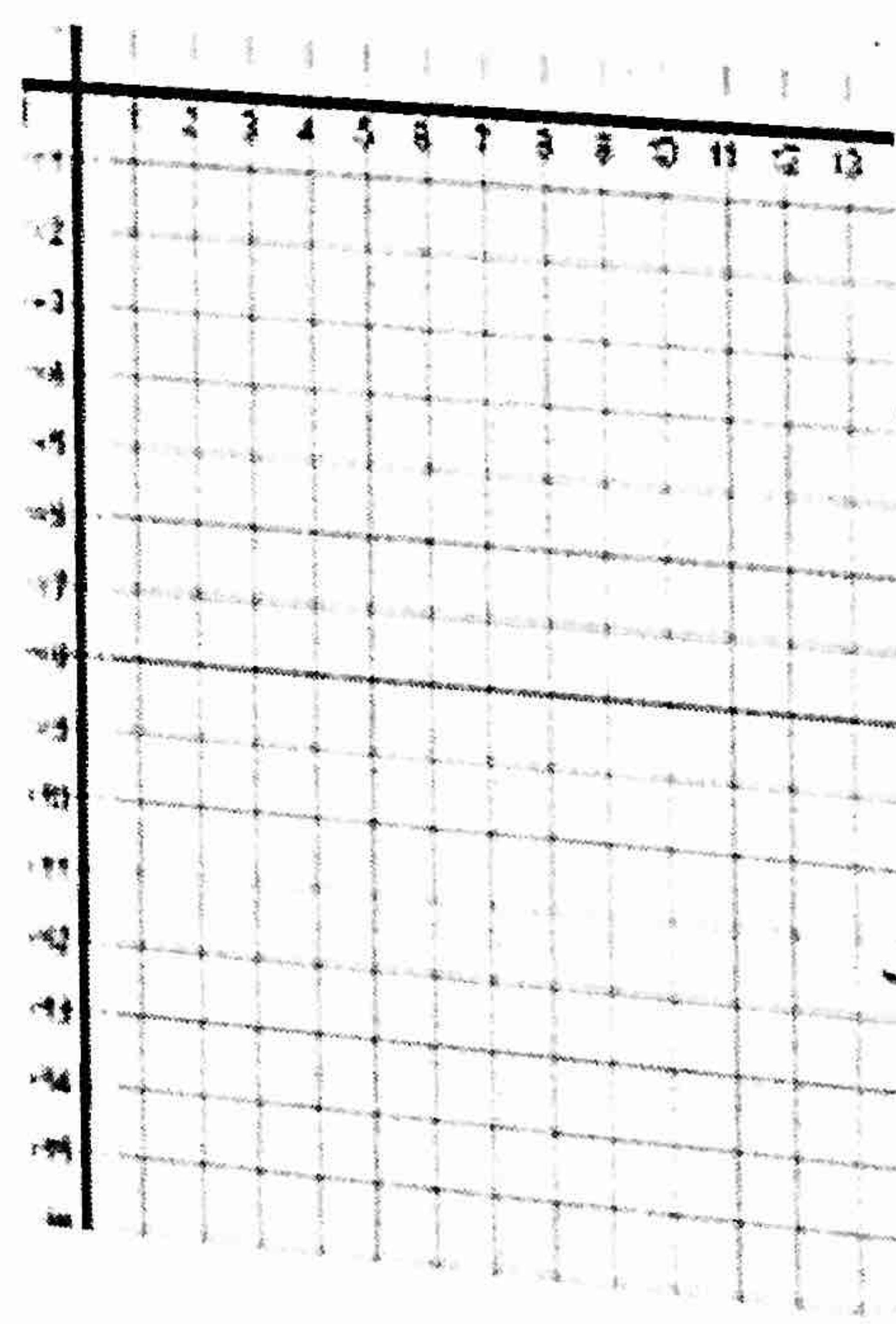
Y-Intercept @ N/A

End behavior

as $x \to \infty, y \to$ ____

as $x \to -\infty, y \to$ ____

25. $f(x) = log(x + 1) - 1$



Domain: $(-1, \infty)$

Range: $(-\infty, \infty)$

Increase or Decrease

Asymptote @ -1

Y-Intercept @ -1

End behavior

as $x \to \infty, y \to$ ____

as $x \to -\infty, y \to$ ____

Unit 2 Learning Target 5: I can find inverses of exponential and logarithmic functions. (___/15)

Given each function below, find the inverse function.

26. $y = 13^x$

$x = 13^y$

$\boxed{\log_{13}(x) = y}$

27. $g(x) = 5^x - 7$

$\boxed{\log_5(x+7) = y}$

$y = 5^x - 7$

$x = 5^y - 7$

$+7 \quad +7$

$x + 7 = 5^y$

28. $h(x) = 7 \cdot 15^{x+8}$

$y = 7 \cdot 15^{x+8}$

$\frac{x}{7} = \frac{7 \cdot 15^{y+8}}{7}$

$\frac{x}{7} = 15^{y+8}$

$\log_{15}\left(\frac{x}{7}\right) = y + 8$

$-8 \quad -8$

$\boxed{\log_{15}\left(\frac{x}{7}\right) - 8 = y}$

29. $f(x) = \log_4(x)$

$y = \log_4(x)$

$x = \log_4(y)$

$\boxed{4^x = y}$

30. $y = \log_2(x + 2)$

$x = \log_2(y+2)$

$2^x = y + 2$

$\boxed{2^x - 2 = y}$

31. $y = 3\log_7(x - 2)$

$\frac{x}{3} = \frac{3\log_7(y-2)}{3}$

$\frac{x}{3} = \log_7(y-2)$

$7^{\frac{x}{3}} = y - \cancel{2}$

$+2 \quad +2$

$\boxed{7^{\frac{x}{3}} - 2 = y}$